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Infrared singularities in interface growth models

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We show that nonlinear interface growth models with roughness exponent $\alpha \geqslant 1$ have intrinsic nonperturbative infrared singularities which are inaccessible to the usual dynamical renormalization group analysis. We argue that these infrared singularities give rise to a strong-coupling problem in 1+1 dimensions and provide the underlying reason for the difference between local and global dynamic scaling in these models.

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Much recent interest [1] has focused on generic scale invariance aspects of conserved interface growth dynamics where surface diffusion is the dominant relaxation mechanism. It has been argued [1–3] that if surface overhang and/or bulk vacancy formation is negligible, then nonequilibrium surface growth dynamics may be described by a fourth-order conserved equation, introduced by Lai and Das Sarma [2] and by Villain [3], which is distinct from the well-studied [4] generic second-order growth model. In this paper we show that the nonlinear fourth-order conserved growth equation has infrared singularities in 1+1 dimensions akin to those in the fluid turbulence problem. These infrared singularities nontrivially affect the critical growth exponents as derived by dynamical renormalization group (DRG) techniques, giving rise to anomalous dynamic scaling [5–7] and signatures of multiaffinity [7]. Effectively, these infrared singularities convert the problem to a "strong-coupling" problem in the substrate dimension $d=1$, and a perturbative DRG analysis fails to give the "correct" exponents. We propose these infrared singularities underlying the fourth-order growth equation as the reason for a number of recently reported peculiar findings in the computer simulation of surface diffusion dominated conserved discrete growth models. Theoretical results presented here show that the conserved nonlinear MBE growth equation has a type of "lower critical dimensionality" (d_c), $d_c=1$, where the global exponents are still defined by the perturbative DRG theory [2], but significant strong-coupling corrections (not captured in the DRG analysis) arise in $d \leqslant d_c$ leading to anomalous [5,6] and multiaffine [7] scaling.

One of the more interesting recent developments in the analysis of growth models is the discovery [5,6] of anomalous dynamic scaling (and the associated difference between local and global scaling) in 1+1 dimensions. While Das Sarma *et al.* [5] considered the height-height correlation, higher-order correlations were numerically analyzed by Krug [7], who also noted the analogy between anomalous dynamic scaling and the phenomenon of intermittency in turbulence. Accumulating numerical data [8–10] have also indicated that while discrete growth models in 1+1 dimensions yield critical exponents quite different from those obtained from the nonlinear continuum model [2] of Lai, Das Sarma, and Villain (LDV), in 2+1 dimensions they yield the same exponents as the continuum model. Further, in trying to integrate the continuum LDV equation numerically in 1+1 dimensions, Tu noted [11] an instability in 1+1 dimensions. In this paper we point out the existence of a type of infrared singularity for $d \leqslant 1$ in a class of growth models described by the fourth-order conserved nonlinear continuum equation. The existence of the singularity (i) explains the origin of the anomalous dynamic scaling [5]; (ii) gives a qualitative explanation for the similarities and differences in the behaviors of discrete and continuum models; and (iii) provides the mechanism behind the intermittent multifractal behavior observed by Krug [7] in the discrete Das Sarma–Tamborenea (DT) model [12].

The observed empirical similarity between the behavior of position dependent correlation functions in the growth model and the problem of intermittency in turbulence gives us indications for an infrared singularity underlying the anomalous dynamic scaling behavior. It is thought that intermittency in turbulence arises from the existence of an infrared singularity and of an infinite number of relevant operators [13]. We have consequently examined both of these issues in the LDV growth model, finding the existence of an infinite set of relevant operators and an infrared singularity which combine to provide an explanation for anomalous dynamic scaling in this class of growth models. The existence of an infinite set of relevant operators in this growth problem has earlier been noted in the literature [2,7,11].

We begin by briefly recalling the continuum LDV model [2]. The height fluctuation variable $h(\vec{x},t)$ grows according to the dynamical equation

$$\partial h/\partial t = -\nu_4 \nabla^4 h + \lambda_2 \nabla^2 (\nabla h)^2 + \eta, \qquad (1)$$

where η is a nonconserved white noise. The d-dimensional vector $\vec{x}$ lies in the substrate. The height-height correlation function is written as $C(\vec{r},t) = \langle h(\vec{x},t_0) h(\vec{x}+\vec{r};t_0+t)\rangle$ and in Fourier space has the standard dynamic scaling form [14]

$$C(k,\omega) = k^{-2\alpha-d-z} F(\omega/k^z). \qquad (2)$$

The exponents z and α are the usual dynamic and roughness exponents, respectively, with z being associated with a characteristic frequency, $\omega_c \sim k^z$, which characterizes the mode relaxation rate. For a system of size L, Eq. (2) implies that the mean square interface width $\langle h^2(x,t)\rangle \sim L^{2\alpha}$ at very long times when the time dependence of the variance disappears. The long-time dynamic scaling behavior of $C(r) \sim r^{2\alpha}$ is also implied by Eq. (2). According to the DRG analysis of Lai and Das Sarma $\alpha=(4-d)/3$, $z=(8+d)/3$ and these exponents are expected [15] to be "exact" (in the perturbative DRG sense) to all loops. Anomalous dynamic scaling [5,6] involves replacing Eq. (2) by the modified scaling ansatz

$$\int_{-\infty}^{\infty} C(k,\omega)d\omega \propto k^{-2\alpha-d} g(k\xi) \tag{3}$$

where for $k\xi \gg 1$, $g(x) \sim x^{-\kappa}$. Standard scaling [14] would imply $g(x)=\text{const}$ identically. The functional dependence of $g(k)$ on k, which immediately requires a length scale ξ, is the anomalous scaling found in Ref. [5]. The characteristic frequency ω_c scales as $\omega_c = k^z f(k\xi)$ where $f(x) \sim x^{\gamma}$ for $x \gg 1$ in the anomalous scaling situation. Here ξ is the characteristic correlation length along the surface which is cut off by the substrate size L for $\xi > L$. The signature of anomalous behavior in growth models is the existence of the functions $g(x)$ and $f(x)$ (and of the exponents κ and γ) which makes the short distance ($r \ll \xi$) behavior different from that of the usual dynamic scaling situation. [In the usual dynamic scaling situation $g(x)$ and $f(x)$ are constants for $x \gg 1$.] The anomalous behavior for the higher-order correlations was formulated by Krug [7] as $\langle [h(x+r,t)-h(x,t)]^q \rangle^{1/q} = r^{\alpha_q} \xi^{\kappa q} f_q(r/\xi)$, with $f_q(x) \to$ const for $x \to 0$, which is an obvious generalization of Eq. (3) for $q \neq 2$. (We note that we have changed the exponent notations of Ref. [7] to conform to ours.) The fact that α_q is a function of q is the signature [16] of multifractal behavior.

Anticipating that part of the anomalous behavior stems from the existence of the relevant operators in Eq. (1), we start with the "higher-order" growth equation

$$\frac{\partial h}{\partial t} = -\nu_4 \nabla^4 h + \nabla^2[\lambda_2(\nabla h)^2 + \lambda_4(\nabla h)^4 + \cdots] + \eta. \tag{4}$$

We take the view that the asymptotic critical properties of the model with $\lambda_4 = \lambda_6 = \cdots = 0$ [i.e., Eq. (1)] have been exactly solved [2,15] in the perturbative DRG sense and the answer expressed by the LDV exponents. What we explore in this work is whether there is a lower critical dimension d_c such that for $d \leq d_c$ all the "higher-order" $\lambda_4, \lambda_6, \ldots,$ etc. terms become "relevant," affecting the DRG exponents [2] in a strong-coupling nonperturbative sense. Power counting shows that for $\alpha < 1$ the couplings λ_{2n} with $2n \geq 4$ are irrelevant, while for $\alpha > 1$ they become relevant with $\alpha = 1$ as the marginal situation. Thus, in $d=1$ (<1) all operators $\lambda_{2n}(\nabla h)^{2n}$ are marginal (relevant). For $d>1$, the higher-order terms are irrelevant.

We now exhibit the vital feature in $d=1$ which makes this the lower critical dimension. To do so we work with the original LDV model, Eq. (1), to begin with and calculate the propagator $G(k,\omega)$ in perturbation theory. As usual

$$G^{-1}(k,\omega) = G_0^{-1}(k,\omega) + \Sigma(k,\omega) = -i\omega + \nu_4 k^4 + \Sigma(k,\omega), \tag{5}$$

so that the physical significance of the self energy $\Sigma(k,\omega)$ is the characteristic relaxation rate, and $\Sigma(k,\omega)$ has the scaling form $\Sigma(k,\omega) = k^z \sigma(\omega/k^z)$ where $\sigma(x) \to$ const as $x \to 0$, so that $\Sigma(k,0) = \Gamma k^z$. The propagator is dominated by $\Sigma(k,\omega)$ when $z<4$, which occurs for $d<4$. In this range, we obtain the single loop self-consistent contribution [Fig. 1(a)]:

$$\Sigma(k,0) = \lambda_2^2 k^2 \int_{\vec{p}+\vec{q}=\vec{k}} \frac{d^d p}{(2\pi)^d} \frac{d\omega}{2\pi} (\vec{k}\cdot\vec{q}) q^2 p^2 C(\vec{p},\omega) G(\vec{q},\omega)$$

$$\sim \lambda_2^2 k^2 \int \frac{d^d p}{(2\pi)^d} \frac{(\vec{k}\cdot\vec{q})q^2}{p^{2\alpha+d-2}} \frac{1}{p^z + q^z}, \tag{6}$$

where in the second line we have made the standard Lorentzian approximation for the shape of the correlation function.

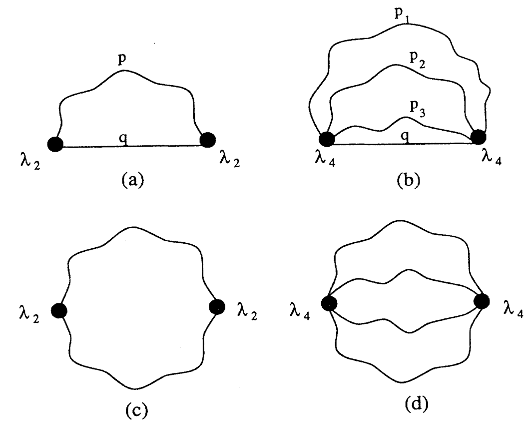

FIG. 1. The two lowest infrared singular contributions to the self-energy [(a) and (b)] and the correlation function [(c) and (d)] arising from the λ_2 [(a) and (c)] and the λ_4 [(b) and (d)] nonlinearities (solid line is the dressed propagator and the wavy line is the dressed correlator or the spectral density function).

Using $\Sigma(k) \sim k^z$, it is immediately clear from Eq. (5) that power counting supports $\alpha + z = 4$. The validity of the power counting depends on the p integral being finite and we find on inspection that as $p \to 0$ the integral behaves as $\int d_p / p^{2\alpha-1}$, which is infrared divergent for $2\alpha - 1 \geq 1$ or $\alpha \geq 1$. As noted previously, this means $d \leq 1$ with $d=1$ being the marginal dimension where the infrared singularity produces a logarithmic divergence. From now on, we concentrate on $d=1$. Using ξ^{-1} as the usual cutoff to suppress the logarithmic divergence in Eq. (6) we get

$$\Sigma(k) \equiv \Sigma(k,0) \sim k^3 \ln(k\xi). \tag{7}$$

An identical argument for the correlation function shows a similar logarithmic correction. Thus the continuum model of Lai and Das Sarma will have logarithmic corrections at $d=1$ as was mentioned in Ref. [5]. Perturbative DRG analysis misses this "strong-coupling" infrared singularity.

We now explore the contributions from the higher-order nonlinearities $\lambda_4, \lambda_6, \ldots,$ etc. which are all marginal in $d=1$. The lowest-order (λ_4) diagram is shown in Fig. 1(b) and it is immediately obvious that when the momentum flows through the propagator line, there will be an infrared singularity with the strength $[\ln(k\xi)]^3$. From the λ_4 term the contribution to $\Sigma(k)$ reads $\sim k^3[\ln(k\xi)]^3$. The result for the λ_{2n} term can be easily envisaged to be $k^3[\ln(k\xi)]^{2n-1}$.

Having obtained an infinite logarithmic series of the form $k^3 \Sigma_n a_n [\ln(k\xi)]^{2n-1}$, where the coefficients a_n depend explicitly on the coupling strengths λ_{2n}, we are unable to proceed any further without making some essential assumptions about the behavior of the strong-coupling series. There are two possibilities: the infinite series does not converge to a universal strong-coupling fixed point, implying nonuniversal model-dependent scaling behavior, or, the coupling constants λ_{2n} are driven to some universal (i.e., model-independent) nonperturbative fixed point, leading to universal strong-coupling critical behavior. Anticipating the existence of a strong-coupling fixed point in the vicinity of which (possibly nonuniversal) scaling is regained, we can exponentiate the singular λ_{2n}-series of logarithms to write

$$\Sigma(k) \sim k^3 (k\xi)^{\gamma}. \tag{8}$$

The anomalous exponent $\gamma(>0)$ is, in principle, generically nonuniversal unless a true strong-coupling fixed point exists which assures the flow of the coupling strengths λ_{2n} to the fixed point. We are, however, unable to calculate γ exactly unless the nature of the nonperturbative fixed point is clarified.

We now turn to the correlation function $C(k,\omega)$ and note that the lowest-order contribution from λ_2 in $d=1$ [Fig. 1(c)] is

$$k^2C(k,\omega)\sim\lambda_2^2|G_0(k,\omega)|^2k^6\int\frac{dp}{p^3q^3}\frac{p^2q^2}{-\frac{i\omega}{\Gamma}+p^3+q^3}. \tag{9}$$

Clearly, there is the same infrared $(p\to 0)$ divergence in the integral in Eq. (9) as we have already discussed, and $\int C(k,\omega)d\omega\sim k^{-3}\ln(k\xi)$. The λ_4 contributions of Fig. 1(d) have a similar infrared divergence as discussed previously and an exact repetition of previous argument for the λ_{2n} series yields

$$\int C(k,\omega)d\omega\sim k^{-3}(k\xi)^{\kappa}, \tag{10}$$

where the anomalous exponent κ differs from γ only by a numerical constant. Once again $\kappa>0$ and in terms of the correlation function in real space this implies that the $r^{2\alpha}$ behavior is replaced by $r^{2\alpha}(r/\xi)^{-\kappa}$. Thus the roughness exponent α gets replaced by α' in the correlation function with $\alpha'=\alpha-\kappa/2$ and the short-distance correlation function develops a time dependence $\xi^{\kappa}\sim t^{\kappa/z}$ for $\xi<L$, which are precisely the anomalous dynamic scaling findings [5] in numerical simulations. Equation (10) establishes anomalous dynamic scaling for the height-height correlation function, but *not* multiscaling of higher-order C_q's.

If we now turn to the higher-order height difference correlators C_q studied by Krug [7], $C_q(r)\sim\langle|h(x,t)-h(x+r,t)|^q\rangle$, then using the correlation function of Eq. (10) and a Gaussian closure approximation, we find *in the lowest-order factoring approximation* that $C_q\sim r^{2\alpha'}$ for all $q\geqslant 2$ where $2\alpha'=2\alpha-\kappa$, to be contrasted with the usual dynamic scaling result $C_q\sim r^{\alpha q}$. This is because in the Gaussian decomposition and in the subsequent integration over the internal momenta, all the integrals save one yield cutoff dependent constants–the scale dependence comes from only one internal line and hence for all $q>2$ one obtains the same answer as $q=2$. At this lowest order, $C_q^{1/q}\sim r^{2\alpha'/q}$ and therefore the multiaffine q dependence of the various moments of the height correlation is rather simple. It is reassuring to see that Krug's numerical results [7] for the DT model are not inconsistent with this simple lowest-order invariance law, i.e., $C_q\sim r^{2\alpha'}$, to within $\pm 25\%$. This is not to say that the deviation from the invariance shown by the numerical data [7] is spurious. Indeed a preliminary investigation of the higher-order diagrams shows that corrections develop which are either nonuniversal or depend on the fixed point values of λ_{2n} $(n\geqslant 2)$, depending on whether a strong-coupling fixed point exists or not.

Finally, we turn to discrete nonequilibrium growth models and discuss the possibility that all the recently introduced [1–3] surface diffusion driven interface growth models which have $\alpha\geqslant 1$ are described by Eq. (4), and are, therefore, infrared singular. We have studied four discrete growth models numerically (Fig. 2) which we find are well described by

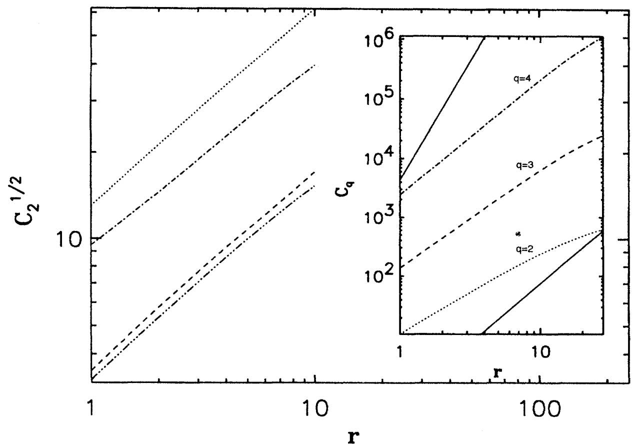

FIG. 2. The square root of the numerically calculated long-time steady-state height-height correlation function $C_2(r)$ (system size $=500$) for four discrete nonequilibrium growth models. Top to bottom: $\alpha'=0.68$ (DT); 0.63 (modified DT); 0.68 (DT with diffusion length $=3$); 0.67 (LD). Inset shows the long-time $C_q(r)$ with $q=2,3,4$ (bottom to top) for the discrete LD model which has $\alpha=1$ (the slopes $2\alpha=2$ and $4\alpha=4$ are also shown for comparison); similar results for the DT model are given in Ref. [7].

the phenomenology discussed in this paper. These are the DT model [12], the discrete LD model [2], the DT model with a finite diffusion length [1], and a modified DT model where after each instantaneous deposition or diffusion event all singly bonded particles at the growth front which have available nearest-neighbor kink sites are allowed to move there. In Fig. 2 we show our calculated long-time $C_2(r)$ in $d=1$ for these discrete models, finding that all of them have approximately the same value of $\alpha'=\alpha-\kappa/2\approx 0.7$. Our calculated results for the higher moments of the height difference correlation function with $q=3,4$ (shown as an inset in Fig. 2 for the discrete LD model) in these models also approximately (within $\pm 20\%$) satisfy the lowest-order theoretical result $C_q\sim r^{2\alpha'}$. The significance of the fact that these four different models which have very different values of effective $\alpha=1.4$ (DT), 1.3 (DT with finite diffusion length), 1.6 (modified DT), 1.0 (LD) as obtained from their saturated interface width scaling, all have the same anomalous exponent α' is not clear at this stage. The four models are characterized by very different values of the multiscaling exponent κ (such that $\alpha'=\alpha-\kappa/2$ is approximately a constant). It is quite clear, however, that the four models approximately obey the phenomenology discussed in this paper, and *for a given model* the infrared singular behavior defined by Eqs. (8) and (10) is valid, even though the values of the anomalous exponents γ and κ seem to be nonuniversal among the four models. This, in fact, is quite plausible if we accept that each discrete model follows Eq. (4) in the coarse-grained continuum limit with its own unique set of coupling strengths λ_{2n} which varies from model to model, and there is no universal strong-coupling fixed point for all possible discrete models obeying Eq. (4), implying nonuniversal values of the multifractal exponent κ in different models. In fact, there is already some numerical evidence for the nonuniversal nature of anomalous dynamic scaling [17].

To conclude, we have carried out a leading-order mode-coupling analysis of the Lai–Das Sarma–Villain nonlinear growth equation [Eq. (1)] as well as its natural higher-order nonlinear generalization [Eq. (4)] in order to investigate whether the anomalous and multiaffine scaling discovered numerically [5–7] in super-rough discrete growth models (with the global roughness exponent $\alpha \geqslant 1$) could arise from infrared singularities inherent in the LDV growth equation. We find that there are nonperturbative infrared singularities for $d \leqslant d_c$ with $d_c = 1$ as the lower critical dimensionality of the problem where the infrared singularities produce nonuniversal logarithmic power series corrections to the DRG growth exponents [2]. The failure of the perturbative DRG theory for $d \leqslant d_c$ is signaled by the calculated [2] DRG roughness exponent $\alpha = (4-d)/3$ with $\alpha \geqslant 1$ for $d \leqslant d_c = 1$, which implies that $|\nabla h|$ diverges asymptotically and, therefore, all the higher-order nonlinear corrections of the form $\nabla^2(\nabla h)^{2n}$ [cf. Eq. (4)] become marginal (relevant) for $d = d_c$ ($< d_c$) with respect to the LDV nonlinearity $\nabla^2(\nabla h)^2$. Our mode-coupling analysis indicates that the *local* roughness exponent α' characterizing the height-height correlation function is affected by the infrared singularity in $d = 1$ leading to $\alpha' = \alpha - \kappa/2$, where κ is a (possibly) nonuniversal scaling exponent associated with the anomalous scaling in the problem. Thus the infrared singularity arising from asymptotically divergent $|\nabla h|$ leads to a difference between local and global scaling in the kinetic roughening problem, as discovered [5,7] in the numerical simulations of discrete super-rough kinetic growth models. In some sense, the higher-order nonlinear terms of the form $\nabla^2(\nabla h)^{2n}$ with $n > 1$ in Eq. (4) act in $d = 1$ as dangerously irrelevant fields with a formal similarity to the corresponding dangerous irrelevant variable situation in the static critical phenomena [18]. The very interesting point is that the "dangerous irrelevant field" in this problem changes the *local* (but *not* the global) roughness exponent as well as the scaling function whereas in the static critical phenomena the dangerous irrelevant variable affects only the asymptotic scaling function. We know of no situation in static critical phenomena where a similar strong-coupling anomalous and multiaffine scaling situation arise due to nonlinear infrared singularities. It is conceivable that a similar scenario may apply to the *equilibrium* roughening problem [19] in $d \leqslant 0$ (i.e., $d_c = 0$) where the corresponding wandering exponent $\zeta(d) = \frac{1}{2}(2-d)$ becomes $\zeta(d=0) = 1$. There have been very few studies [20] of the equilibrium roughening problem around $d = 0$, and to the best of our knowledge the issue of anomalous and multiaffine scaling behavior has never been discussed in the context of the equilibrium roughening phenomena [21]. Finally, one could legitimately question our use of the mode-coupling technique to study the strong-coupling LDV behavior. We believe that the mode-coupling analysis is adequate to establish the existence of the strong-coupling infrared singular behavior of Eqs. (1) and (4) for $d \leqslant d_c$. Whether the mode-coupling technique can produce the correct strong-coupling exponents for a particular model is always a problematic issue and we have nothing to add to this formal theoretical question. We can only state that the mode-coupling technique has recently been extensively used [22] in the kinetic roughening problem to study the strong-coupling behavior of the Kardar-Parisi-Zhang equation, and our use of it in this paper for the LDV equation, while being somewhat speculative, should be of the same level of validity. One specific prediction is that there should be no anomalous scaling and/or multiaffine scaling in growth models with $\alpha < 1$. Thus in DT type models, our theory specifically rules out anomalous scaling and multiscaling in $d = 2 > d_c = 1$. We propose that extensive simulations be carried out to check whether there is anomalous scaling in $d = 2$ DT models, which will be a direct test of the infrared singular mode-coupling theory developed in this paper.

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